## QUIZ 5 SOLUTIONS: LESSONS 5-6 SEPTEMBER 8, 2017

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Evaluate  $\int x^3 \ln x \, dx$ 

<u>Solution</u>: This is an integration by parts problem. By LIATE, take  $u = \ln x$ , then  $dv = x^3 dx$ . Our table becomes

$$u = \ln x \qquad dv = x^3 dx$$
$$du = \frac{1}{x} dx \qquad v = \frac{1}{4} x^4$$

By integration by parts formula,

$$\int x^{3} \ln x \, dx = \underbrace{\ln x}_{u} \underbrace{\left(\frac{1}{4}x^{4}\right)}_{v} - \int \underbrace{\left(\frac{1}{4}x^{4}\right)}_{v} \underbrace{\left(\frac{1}{x}\,dx\right)}_{du}$$
$$= \frac{1}{4}x^{4} \ln x - \frac{1}{4} \int x^{3} \, dx$$
$$= \frac{1}{4}x^{4} \ln x - \frac{1}{4} \left(\frac{1}{4}\right)x^{4} + C$$
$$= \boxed{\frac{1}{4}x^{4} \ln x - \frac{1}{16}x^{4} + C}$$

2. [5 pts] Find a particular solution to  $\frac{dy}{dx} = \frac{x}{y^2}$  if y = 1 when x = 0.

Solution: Write

$$\frac{dy}{dx} = \frac{x}{y^2}$$

$$\Rightarrow \quad y^2 \frac{dy}{dx} = x$$

$$\Rightarrow \quad y^2 \, dy = x \, dx$$

$$\Rightarrow \quad \int y^2 \, dy = \int x \, dx$$

$$\Rightarrow \quad \frac{1}{3} y^3 = \frac{1}{2} x^2 + C$$

We know y = 1 when x = 0, so

$$\frac{1}{3}(1)^3 = \frac{1}{2}(0)^2 + C \Rightarrow C = \frac{1}{3}.$$

Hence,

$$\frac{1}{3}y^3 = \frac{1}{2}x^2 + \frac{1}{3}$$
$$\Rightarrow \quad y^3 = \frac{3}{2}x^2 + 1$$
$$\Rightarrow \quad y = \sqrt[3]{\frac{3}{2}x^2 + 1}$$

Therefore, the particular solution is

$$y = \sqrt[3]{\frac{3}{2}x^2 + 1}$$

Bonus: This question is worth up to 1 point towards a max total of 10 points.

Suppose a population of bacteria grows at a rate k proportional to its population, P(t). Write down a formula for P'(t).

**Solution**: P'(t) = kP(t)

The key wording here is that the population is growing at a rate proportional to its population.